LITERATURE CITED

1. E. A. Novikov, "Dynamics and statistics of a system of vortices," Zh. Eksp. Teor. Fiz., 68, No. 5 (1975).
2. S. Kida, "Statistics of a system of line vortices," J. Phys. Soc. Jpn., 39, No. 5 (1975).
3. Y. B. Pointin and T. S. Lundgren, "Statistical mechanics of two-dimensional vortices in a bounded container," Phys. Fluids, 19, No. 10 (1976).
4. Y. B. Pointin and T. S. Lundgren, "Statistical mechanics of two-dimensional vortices," J. Stat. Phys., 17, No. 5 (1977).
5. A. J. Chorin, "Numerical study of slightly viscous flow," J. Fluid Mech., 57, No. 4 (1973).
6. H. Lamb, Hydrodynamics, Dover (1932).
7. N. N. Bogolyubov, Problems of Dynamical Theory in Statistical Physics, Collected Works, Vol. 2 [in Russian], Naukova Dumka, Kiev (1970).
8. R. Balescu, Statistical Mechanics of Charged Particles, Krieger (1963).
9. R. Balescu and A. Senatorski, "A new approach to the theory of fully developed turbulence," Ann. Phys. 58, No. 2 (1970).
10. P. Resibois and M. DeLeener, "Irreversibility in Heisenberg spin systems I," Phys. Rev., 152, No. 1 (1966).
11. G. K. Batchelor, Introduction to Fluid Dynamics, Cambridge Univ. Press (1967).
12. N. Rostoker and M. N. Rosenbluth, "Test particles in a completely ionized plasma," Phys. Fluids, 3, No. 1 (1960).
13. R. H. Kraichnan, "Statistical dynamics of two-dimensional flow," J. Fluid Mech., 67, Part 1 (1975).
14. I. Prigozhin, Irreversible Statistical Mechanics [in Russian] Mir, Moscow (1964).
15. P. Resibois, "New approach to irreversible transport phenomena in plasma dynamics," Phys. Fluids, 6, No. 6 (1963).
16. G. A. Korn and T. M. Korn, Manual of Mathematics, McGraw-Hill (1967).
17. I. S. Gradshteyn and I. M. Ryzhik, Tables of Integrals, Series, and Products, Academic Press, New York (1965).
18. G. Joyce and D. Montgomery, "Negative temperature states for the two-dimensional guiding-center plasma," J. Plasma Phys., 10, 1 (1973).
19. V. P. Starr, Physics of Negative Viscosity Phenomena, McGraw-Hill (1968).
20. R. Balescu, Equilibrium and Non-Equilibrium Statistical Mechanics, Wiley (1975).

STATISTICAL PROPERTIES OF BURSTS OF TURBULENT FLUCTUATIONS

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UDC 532.517.4

The energy spectra, probability distribution functions (DFs), and the associated moment and scale numerical characteristics are used to describe turbulent fluctuations at a certain point of a flow in statistical fluid mechanics. However, these functions do not describe the instantaneous disturbances generated in turbulent flows; these disturbances are particularly important in a number of engineering applications.

An alternative approach to the investigation of turbulence is possible, consisting in the analysis of bursts, i.e., events where the fluctuation component of the flow velocity exceeds a certain prescribed level. Apart from practical applications, the burst characteristics determined by the joint distribution of the probabilities of the fluctuation velocity of the flow and its derivative are important from the standpoint of methods being developed at the present time for the description of turbulent flows on the basis of the DF equations. This kind of approach can be used in studying the laminar-to-turbulent flow transition, which is characterized by the inception of randomly distributed local regions with large gradients of the parameters.

The theory of bursts of stochastic processes, which was formulated primarily for radiophysical applications (see, e.g., [1]), is cur...ly in a state of continuing development.

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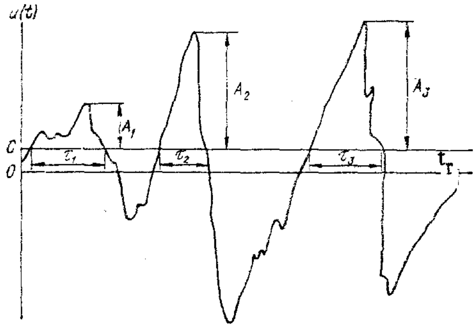

Fig. 1

The existing theoretical results are inadequate for the quantitative description of instantaneous disturbances in turbulent flows, but they are useful for the qualitative analysis of the problems in question. Scattered experimental data on the parameters of bursts in turbulent flows may be found in [2-4].

The objective of the present study is to determine the functional form of the burst characteristics and to deduce their dependence on the standard parameters of flow fluctuations (rms level, integral scale, etc.) over a broad range of variation of the turbulence Reynolds number.

Let u(t) (t is the time) be the longitudinal component of the fluctuation velocity of the flow. Figure 1 shows a realization of the process u(t) of duration t_r. We define a burst as an excess of the signal u(t) above a prescribed level C. The durations τ_i, the amplitudes A_i, and the number of bursts N (for the realization shown in Fig. 1, N = 3) are random variables. From the burst characteristics τ_i, A_i (i = 1, 2, ..., N) we can form the DF $\Phi_\varphi(\Psi)$ (where $\varphi = \tau$, A) and also determine the average number of bursts per unit time $n = N/t_r$, the average values $m_\varphi = \langle\varphi_i\rangle$, and the rms values $\sigma_\varphi = \sqrt{\langle(\varphi_i - m_\varphi)^2\rangle}$ of the durations and amplitudes (the angle brackets denote averaging over the sample or the time t).

We have determined the burst characteristics for seven realizations. The first four realizations represented fluctuations of the longitudinal component of the flow velocity in the symmetry plane of the wake of a circular cylinder of diameter d = 36 mm set up at the exit orifice of the nozzle of a wind tunnel with a diameter of 1200 mm. The values of the exit velocities U_0 of the air from the nozzle, the Reynolds number $Re = U_0 d/\nu$, and the average flow velocities U at the measurement point are given in Table 1. The measurements were performed using a Disa Elektronik (Denmark) 55A01 constant-temperature hot-wire anemometer with a 55A22 sensor (platinized tungsten wire with a diameter of 5 μm and length of 1 mm). The fifth and sixth realizations were fluctuations of the total pressure p(t) in the exit section of an air-intake model. It is readily shown (see, e.g., [5]) that if fluctuations of the static pressure and density are neglected, then for low intensities of the velocity fluctuations the quantities p(t) and u(t) at a fixed point of the flow are joined by the linear relation $p(t) = \rho U u(t)$, where ρ is the density of the gas. The mass-flow average (bulk) values of the flow velocities U_0 in the model duct, the Reynolds numbers $Re = U_0 D/\nu$ (D is the diameter of the duct in the exit section), and the average velocities U at the measurement point are given in Table 1. The total-pressure fluctuations were measured with a DMI-II-0.6 sensor connected to a 4-ANCh-22 amplifier. The seventh realization was the output signal of a G2-37 normal white-noise generator after transmission through an RC low-pass filter (R = 3 kΩ, C = 1 μF). Realizations 1-7 were recorded on an FM magnetograph (MR 800 A Labcorder) in the frequency range 0-5 kHz. The duration of each frame of the records was 45 sec for the velocity fluctuations, 30 sec for the pressure fluctuations, and 1 min for the normal noise.

The signals were processed on a digital computer. Each realization was first analyzed in the frequency range 0-5 kHz at the interrogation frequency of the analog-to-digital converter f_0 = 20 kHz. The rms level u' of the velocity fluctuations and the integral time scale T were determined, and the upper frequency limit f_u of energy-carrying frequencies was calculated from T in accordance with the condition $2\pi T f_u \approx 5$ (the number 5 is largely an arbitrary choice insofar as the concept of "energy-carrying frequencies" is not strictly defined; according to published data [6], approximately 80% of the energy of turbulence is concentrated in the frequency range up to $f_u = 5/2\pi T$). The values of the intensities of the

TABLE 1

Realization No.	1	2	3	4	5	6	7
U_0, m/sec	5,24	10,5	25,5	51,5	137	241	—
$\mathrm{Re}\cdot 10^{-5}$	0,126	0,253	0,613	1,24	5,24	9,22	—
U, m/sec	4,08	8,64	20,8	42,9	133	200	—
u'/U, %	11,1	10,4	9,88	9,32	4,20	6,06	—
T, msec	9,12	6,77	5,23	2,71	0,163	0,395	3,02
$\mathrm{Re}_T\cdot 10^{-4}$	0,112	0,350	1,50	3,10	0,815	6,35	—
f_u, Hz	100	125	160	315	5000	2000	250
$S\cdot 10^2$	11,4	5,05	−1,32	−6,26	−31,4	−29,6	−0,18
$E\cdot 10^2$	40,0	−7,79	−4,25	−18,2	−28,8	−10,1	1,08
λ, msec	6,87	4,58	4,11	1,97	0,0983	0,252	2,68
SNR, dB	45	45	45	44	29	32	42
Points in Figs. 2—6	5	4	3	2	1	6	7

velocity fluctuations, the integral time scales, the turbulence Reynolds numbers $\mathrm{Re}_T = u'L/\nu$ ($L = TU$), and the frequency limits f_u are given in Table 1.

The signals were subsequently processed in the frequency range from 1 Hz to f_u using filters with a steep response: 48 dB/octave. At the energy-carrying frequencies for realizations 1-7 we calculated the rms levels $\sigma = \sqrt{\langle u^2(t)\rangle}$, the skewness parameters $S = \langle u^3(t)\rangle/\sigma^3$, the kurtoses $E = \langle u^4(t)\rangle/\sigma^4 - 3$, the DFs, and the quantities $\lambda = \sqrt{2\langle u^2(t)\rangle/\langle(\partial u/\partial t)^2\rangle}$ (since the analysis was carried out at the energy-carrying frequencies obtained in the experiments, the characteristic times λ of the processes should not be identified with the time microscales of Taylor). The values of the parameters S, E, λ, along with the signal-to-noise ratio (SNR) of the instruments at the output of the magnetograph in the frequency range from 1 Hz to f_u are given in Table 1.

The burst characteristics for each value of the level C were calculated from segments of realizations of sufficient length to ensure a sample size N = 512. The relative statistical errors of determination of the quantities m_φ and σ_φ in this case were equal to 4.4% and 6.3% respectively. A special analysis has shown that for the correct determination of the functions $\Phi_\varphi(\varphi)$ the interrogation frequency must satisfy the condition $f_0 m_\tau \geqslant 50$. In the reported experiments the frequency f_0 was varied in accordance with this level from 5 to 800 kHz.

The experiments showed that for realizations 1-6 the DFs of the flow fluctuations are almost normal. Clearly, the DF for realization 7 is Gaussian. It is reasonable to expect, therefore, that the theoretical results for normal stochastic processes will be applicable to the investigated realizations.

The results of the measurements lead to the conclusion that the DFs of the durations of the bursts of the flow velocity fluctuations for $c = C/\sigma \leqslant 1.5$ obey a log-normal law. As an illustration, Fig. 2 shows some of the experimental data in probabilistic scale. The graphs are plotted as follows. The abscissas (suitably shifted) represent the values of the quantity $z = (x - m_x)/\sigma_x$, where $x = \ln(\tau/\lambda)$, $m_x = \langle x_i\rangle$, $\sigma_x^2 = \langle(x_i - m_x)^2\rangle$, and the ordinates represent the linear scale of the numbers z and the corresponding values of the probability integral

$$\Phi_0(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\frac{t^2}{2}}\, dt. \qquad (1)$$

In Fig. 2 the solid lines represent relation (1). The numerals designate the following measurement conditions: 1) c = 2.5, z = y; 2) c = 2.0, z = y − 1; 3) c = 1.5, z = y − 2; 4) c = 1.0, z = y − 3; 5) c = 0.5, z = y − 4; 6) c = 0, z = y − 5; 7) c = 0, z = y − 6.

The nomenclature for the different realizations are given in Table 1. Significant deviations of the burst-duration DFs from the law (1) for realizations 1-6 appear for $c \geqslant 2$. For

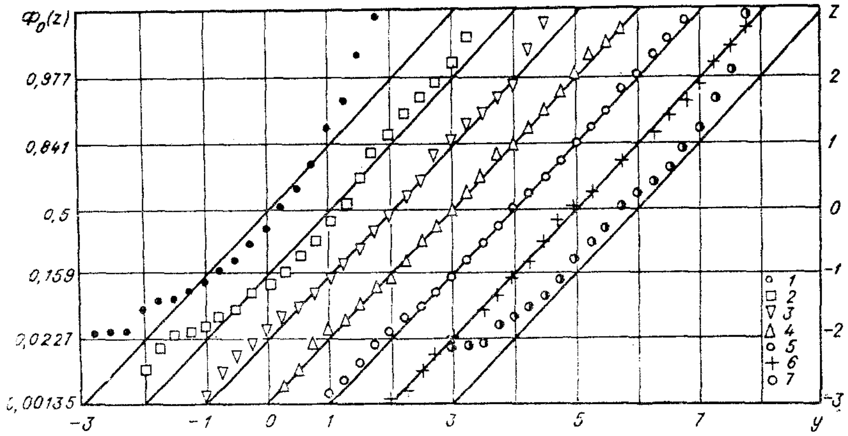

Fig. 2

normal noise, however, the burst-duration DFs, even for c = 0, are log-normal (see Fig. 2). A log-normal distribution of the durations of the bursts of turbulent fluctuations at the energy-carrying frequencies has been recorded in a study [3] of the flow velocity fluctuations in a boundary layer, in a planar jet, and behind grids for c = 0, and also of the concentration fluctuations of a passive admixture in entrained and submerged circular jets for small values of c (see [2]). This effect is not a general property of normal stochastic processes, but is associated with the physical mechanism of the evolution of developed turbulence and admits the following qualitative interpretation. It has been shown [7] that a log-normal distribution corresponds asymptotically to the size distribution of particles obtained as a result of successive independent comminutions (the length scale l of instantaneous turbulent formations is linearly related to τ by the equation $l = \tau U$). The log-normal distribution of the burst durations confirms the fact that the physical model of the cascade process of formation of increasingly smaller turbulent eddies at the energy-carrying frequencies is a process of successive independent comminution (breakup) of large eddies (see, e.g., [8]).

It is generally known (see, e.g., [1]) that for large values of c the DFs of the durations of bursts of normal stochastic processes tend to a Rayleigh law, which in our notation can be written in the form

$$\Phi_\tau(\tau) = 1 - \exp\left[-\frac{\pi}{4}\left(\frac{\tau}{m_\tau}\right)^2\right]. \tag{2}$$

The measurements show that relation (2) is applicable for $c \geqslant 2$, and the correspondence of the experimental data to relation (2) improves with increasing value of c. To illustrate this assertion Fig. 3 shows in probabilistic scale some results of measurements of the DFs of burst durations at high intensity levels. The solid lines represent relation (2). The numerals represent the following measurement conditions: 1) c = 1.9, $\tau/m_\tau = y$; 2) c = 2.1, $\tau/m_\tau = y - 0.5$; 3) c = 2.3, $\tau/m_\tau = y - 1$; 4) c = 2.5, $\tau/m_\tau = y - 1.5$.

We now discuss the DFs of the burst amplitudes. It is well known (see, e.g., [1]) that the maxima of wideband normal stochastic processes have a normal distribution. Although the amplitudes of the disturbances are not identical with the maxima involved in the theory of bursts, it is reasonable to expect that the quantities A_i representing the discrete values of a normal random function will also have a Gaussian distribution function. For the absolute values of the amplitudes (considering $c \geqslant 0$) we write the normal DF in the form

$$\Phi_A(A) = 2\Phi_0\left(\sqrt{\frac{2}{\pi}}\frac{A}{m_A}\right) - 1. \tag{3}$$

The results shown in Fig. 4 in probabilistic scale from measurements of the DFs of the burst amplitudes are well described by expression (3) for $0 \leqslant c \leqslant 2.5$. The solid lines represent relation (3). The numerals correspond to the following measurement conditions: 1) c = 2.5, $A/m_A = y$; 2) c = 2.0, $A/m_A = y - 1$; 3) c = 1.5, $A/m_A = y - 2$; 4) c = 1.0, $A/m_A = y - 3$; 5) c = 0.5, $A/m_A = y - 4$; 6) c = 0, $A/m_A = y - 5$; 7) c = 1.5, $A/m_A = y - 6$. A normal DF of the amplitudes of bursts of flow velocity fluctuations has been obtained [3] in a boundary layer, in a planar jet, and behind grids for c = 0. The correspondence of the DFs of the burst amplitudes of normal noise to the distribution (3) is a natural result.

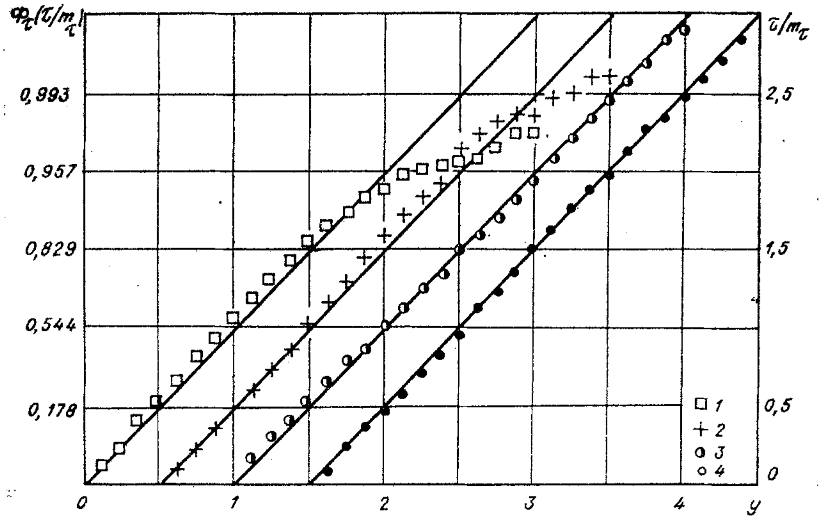

Fig. 3

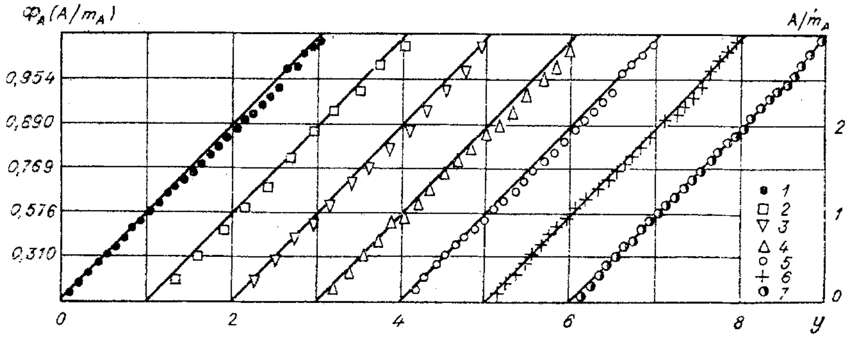

Fig. 4

The universality observed in the reported experiments between the DFs of the durations and amplitudes of bursts of the flow velocity fluctuations at the energy-carrying frequencies greatly simplifies the determination of the dependence of the burst parameters on the standard characteristics of turbulence. For this purpose it is sufficient to relate the governing moments of the resulting distributions to the standard parameters of the fluctuations at various levels.

The log-normal law describing the DFs of the burst durations for $c \leqslant 1.5$ is determined by the quantities m_x and σ_x. It is well known (see, e.g., [9]) that for stochastic processes with a log-normal probability distribution the quantities m_x and σ_x are related one-to-one with the parameters m_τ and σ_τ. In our notation this relationship has the form

$$m_x = \ln \frac{m_\tau/\lambda}{\sqrt{1+(\sigma_\tau/m_\tau)^2}}, \quad \sigma_x^2 = \ln\left[1+\left(\frac{\sigma_\tau}{m_\tau}\right)^2\right]. \tag{4}$$

In Fig. 5 the results of measurements of the average burst duration are compared with the theoretical relation for normal stochastic processes (see [1]):

$$m_\tau/\lambda = \sqrt{2}\pi\,[1-\Phi_0(c)]\,e^{(1/2)c^2}. \tag{5}$$

Equation (5) well describes the experimental data for $c \leqslant 2.1$.

The theoretical relationship of the rms levels of the burst durations to the turbulence level is not to be found in the literature. The measurement data shown in Fig. 5 for $c \leqslant 2.1$ can be approximated by an expression of the form

$$\sigma_\tau/\lambda = \alpha e^{-\beta c}, \tag{6}$$

where α and β are empirical coefficients. The values $\alpha = 2.48$ and $\beta = 0.868$ have been determined by the least-squares method for $c \leqslant 2.1$ from the experimental data given in Fig. 5 for realizations 1-6. It is seen that relation (6) is inapplicable for normal noise.

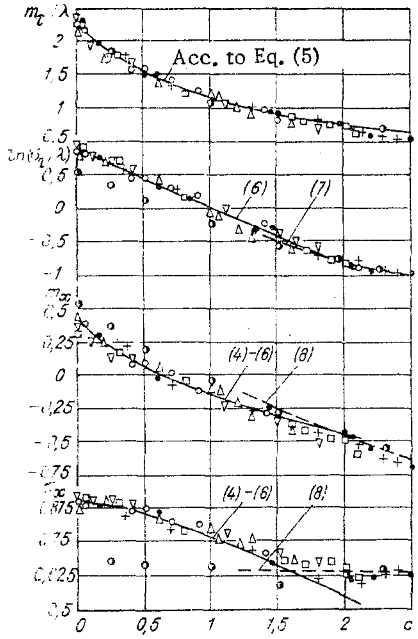

Fig. 5

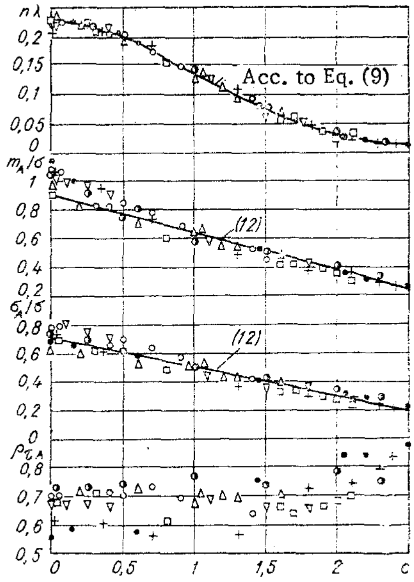

Fig. 6

The results of calculations of the average values m_x and rms values σ_x of the logarithms of the burst durations from relations (4)-(6) for realizations 1-6 are in satisfactory agreement with the measurement data for $c \leqslant 1.5$ (see Fig. 5), thereby supporting the conclusion of a log-normal distribution of the durations of bursts of turbulent fluctuations at low levels. The following approximate estimates have been obtained [3] for the investigated turbulent flows for c = 0: $\langle \lg (\tau/m_\tau)\rangle \simeq -0.26$, $\sqrt{\langle(\lg(\tau/m_\tau) - \langle \lg (\tau/m_\tau)\rangle)^2\rangle} \approx 0.37$. From relations (4)-(6) for c = 0 we obtain for the values of these parameters −0.18 and 0.39 respectively, which are in good agreement with the results of [3] and indicate that relations (5) and (6) clearly have a very general nature.

At high turbulence levels the distribution of the burst durations obeys the Rayleigh law (2), for which $\sigma_\tau/m_\tau = \sqrt{4/\pi - 1}$. For c >> 1, restricting the series expansion of the function $\Phi_0(c)$ to the first two terms, from (5) we obtain $m_\tau/\lambda = \sqrt{\pi/c}$, so that

$$\sigma_\tau/\lambda = (\sqrt{4-\pi})/c. \tag{7}$$

The results of calculations of σ_τ according to (7) are in satisfactory agreement with the measurement data for $c \geqslant 1.8$ for all of the investigated realizations (see Fig. 5). Using the familiar rules for the change of variables in DFs, from (2) we obtain estimates of m_x and σ_x for large c:

$$m_x = \ln \frac{2}{\sqrt{\pi}} - \frac{1}{2} C_e + \ln \frac{m_\tau}{\lambda}, \quad \sigma_x = \frac{\pi}{2\sqrt{6}}, \tag{8}$$

where C_e = 0.5772... is the Euler constant. The satisfactory correspondence of the results of calculations of the parameters m_x and σ_x according to (8) with the measurement data for $c \geqslant 2$ (see Fig. 5) confirms the validity of the distribution (2) for high levels in the case of realizations 1-7.

The results of measurements of the average number of bursts per unit time, shown in Fig. 6, exhibit good agreement with the theoretical relation for normal stochastic processes (see [1]):

$$n\lambda = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}c^2}. \tag{9}$$

The measured values reported in [4] for the average zero-crossing frequency of the fluctuation component of the flow velocity in a circular jet, in a boundary layer, and behind grids show that for c = 0 the group $n\lambda$ is practically independent of the type of flow or the value of Re_T and is approximately equal to 0.23-0.29, consistent with the experimental data shown in Fig. 6.

We now consider the moment characteristics of the burst amplitudes. For the DF (3) the average and rms values are related by the expression

$$\sigma_A/m_A = \sqrt{\pi/2 - 1}, \tag{10}$$

and so the problem is reduced to determining the dependence of m_A on the level c. We assume that for any $c \geqslant 0$ a certain value of the amplitude A* reasonably close to the maximum is obtained with the same confidence coefficient ζ. This assumption corresponds formally to the equation

$$\Phi_A(A^* - C) = \zeta,$$

whence we obtain

$$\frac{m_A}{\sigma} = \sqrt{\frac{2}{\pi}}\frac{1}{k_A}\left(\frac{A^* - C}{\sigma}\right), \tag{11}$$

where k_A is the solution of the equation $\Phi_0(k_A) = (1 + \zeta)/2$. Expression (11) contains two quantities that have to be determined: the confidence coefficient ζ and the corresponding amplitude value A*. In the analysis of almost-normal stochastic processes it is customary to take the value of the process for $\zeta = 0.9973$ as the maximum amplitude. An analysis of the DFs for realizations 1-7 shows that the values of u/σ corresponding to $\zeta = 0.9973$ vary in the interval 3.2-3.6. Assuming in expression (11) that $A^*/\sigma = 3.4$, $\zeta = 0.9973$ (here $k_A = 3$) and making use of (10), we obtain

$$\frac{m_A}{\sigma} = \sqrt{\frac{2}{\pi}}\frac{3{,}4 - c}{3}, \quad \frac{\sigma_A}{\sigma} = \sqrt{1 - \frac{2}{\pi}}\frac{3{,}4 - c}{3}. \tag{12}$$

The results of measurements of the average and rms values of the burst amplitudes in Fig. 6 show that the variations of the quantities m_A/σ and σ_A/σ with the level c are almost linear and are satisfactorily described by expressions (12). It must be borne in mind that the data of direct measurements of the parameters m_A and σ_A in Fig. 6, were not used in deriving expressions (12).

Figure 6 shows the measured values of the correlation coefficient between the burst durations and amplitudes $\rho\tau A = \langle(\tau - m_\tau)(A - m_A)\rangle/(\sigma_A\sigma_\tau)$. It is seen that $\rho_{\tau A}$ increases with the level c. The result is natural insofar as the shape of the bursts becomes more sharply defined at high levels, and it is reasonable to expect that $\rho_{\tau A} \to 1$ as $c \to A^*/\sigma$. However, it appears impossible to establish the functional form of the dependence of the correlation coefficient on the level on the basis of the existing data. The high values obtained experimentally for the correlation coefficient $\rho_{\tau A} \simeq 0.6$-0.9 indicate the presence of an almost-linear statistical relationship between the burst durations and amplitudes (see also [3]). Consequently, long-duration bursts in the turbulent fluctuations of the flow velocity will generally have large amplitudes.

On the whole, the reported experiments show that at the energy-carrying frequencies the characteristics of bursts in turbulent fluctuations of the flow velocity can be described by fairly simple universal laws over a wide range of Re_T and other parameters of the flow. The relations connecting the average and rms values of the burst durations and amplitudes to the threshold level are found to be universal when the quantities σ and λ are used as typical intensity and time parameters. The quantitative description of all the experimental data requires only three empirical constants: α, β, and A*, and the postulated empirical relations for c = 0 are in good agreement with the data of [3, 4]. This means that in developed turbulence the laws governing the formation of bursts in the flow fluctuations at the energy-carrying frequencies are practically independent of the flow conditions or the turbulence Reynolds number, which in the reported experiments did not vary by more than a factor of 50.

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LITERATURE CITED

1. V. I. Tikhonov, Bursts of Stochastic Processes [in Russian], Nauka, Moscow (1970).
2. Yu. A. Shcherbina, "Amplitude-scale characteristic of turbulent fluctuations," in: Turbulent Flows [in Russian], Nauka, Moscow (1974).
3. M. A. B. Narayanan, "Universal trends observed in the maxima of longitudinal velocity fluctuations and the zero crossings in tubulent flows," AIAA J., 17, No. 5 (1979).
4. R. A. Antonia, H. Q. Danh, and A. Prabhu, "Bursts in turbulent shear flow," Phys. Fluids, 19, No. 11 (1976).
5. V. I. Vasil'ev, N. A. Yudenkov, and V. V. Bogdanov, "Intensity of turbulent flow fluctuations in air intakes," Tr. Tsentr. Aérogidrodin. Inst., No. 1327 (1971).
6. V. L. Zimont, A. A. Praskovskii, and A. G. Taryshkin, "Influence of the restricted frequency range of apparatus on the results of measurements of turbulence parameters," Tr. Tsentr. Aérogidrodin. Inst., No. 2048 (1980).
7. A. N. Kolmogorov, "Log-normal particle-size distribution in comminution," Dokl. Akad. Nauk SSSR, 31, No. 2 (1941).
8. A. S. Monin, and A. M. Yaglom, Statistical Fluid Mechanics, Vol. 2, MIT Press (1975).
9. Yu. V. Prokhorov and Yu. A. Rozanov, Probability Theory [in Russian], Nauka, Moscow (1967).

SPREADING OUT OF A VISCOUS LIQUID OVER A HORIZONTAL SURFACE

G. I. Shapiro

UDC 532.62

The spreading out of a viscous liquid over the surface of a solid body plays an important role in a number of practical problems, for example, in the formation of coatings of solid bodies, in the motion of gas–liquid mixtures and emulsions in capillaries, and in other cases [1]. The motion of a thin film of a viscous liquid over a horizontal surface is caused by the action of gravity and surface tension forces and has much in common with the motion of thin films over an inclined surface, which has been intensively studied for a number of years [2-4]. The transition from an inclined plane to a horizontal one is not trivial, i.e., it does not reduce to the substitution into the final formulas of a slope angle equal to zero. The point is that motion over a horizontal surface is described even in the crudest approximation by a differential equation of higher order.

The problem of the spreading out of a viscous liquid over a horizontal surface has been discussed in the two-dimensional formulation in [5], in which the approximate nonlinear equation for the layer thickness h is obtained as a function of the coordinate x and the time t:

$$h_t = (g/3\nu)(h^3 h_x)_x. \tag{1}$$

Here ν is the kinematic viscosity coefficient and g is the gravitational acceleration. Unfortunately, the effect of surface tension has in fact not been taken into account in [5].

In the opposite limiting case, in which one can neglect the force of gravity in comparison with the surface tension force, the equation for h(x, t) has been obtained in [6] (also only in the two-dimensional formulation):

$$h_t + (\sigma/3\rho\nu)(h^3 h_{xxx})_x = 0, \tag{2}$$

where σ is the surface tension coefficient and ρ is the density of the liquid.

The three-dimensional problem of the motion of a viscous incompressible liquid over a horizontal plane is discussed in this paper with account taken of the gravity and surface tension forces. The slope of the free surface is assumed to be small, and the motion is assumed to be sufficiently slow (creeping) so that one can neglect the inertial forces in comparison with the viscous ones. As will be shown, the Reynolds number does not necessarily have to be small. No restrictions are imposed on variations of the layer thickness h(x, y, t); in particular, h can vanish, as occurs upon the spreading out of a drop.

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